NAG Toolbox for MATLAB

Chapter Introduction

F03 – Determinants

Contents

1 Scope of the Chapter

This chapter is concerned with the calculation of determinants of square matrices.

2 Background to the Problems

The functions in this chapter compute the determinant of a square matrix A. The matrix is first decomposed into triangular factors

$$A = LU.$$

If A is positive-definite, then $U = L^{\mathrm{T}}$, and the determinant is the product of the squares of the diagonal elements of L. Otherwise, the functions in this chapter use the Crout form of the LU decomposition, where U has unit elements on its diagonal. The determinant is then the product of the diagonal elements of L, taking account of possible sign changes due to row interchanges.

To avoid overflow or underflow in the computation of the determinant, some scaling is associated with each multiplication in the product of the relevant diagonal elements. The final value is represented by

$$\det A = d1 \times 2^{d2}$$

where $d2$ is an integer and

$$\tfrac{1}{16} \leq |d1| < 1.$$

Most of the functions of the chapter are based on those published in the book edited by Wilkinson and Reinsch 1971. We are very grateful to the late Dr J H Wilkinson FRS for his help and interest during the implementation of this chapter of the Library.

3 Recommendations on Choice and Use of Available Functions

It is extremely wasteful of computer time and storage to use an inappropriate function, for example to use a function requiring a complex matrix when A is real. Most programmers will know whether their matrix is real or complex, but may be less certain whether or not a real symmetric matrix A is positive-definite, i.e., all eigenvalues of $A > 0$. A real symmetric matrix A not known to be positive-definite must be treated as a general real matrix. In all other cases either the band function or the general functions must be used.

The functions in this chapter fall into two easily defined categories.

(i) **Black Box Functions**

These should be used if only the determinant is required. The scaled representation $d1 \times 2^{d2}$ is evaluated as a floating-point number and a failure is indicated if the floating-point number is outside the range of the machine.

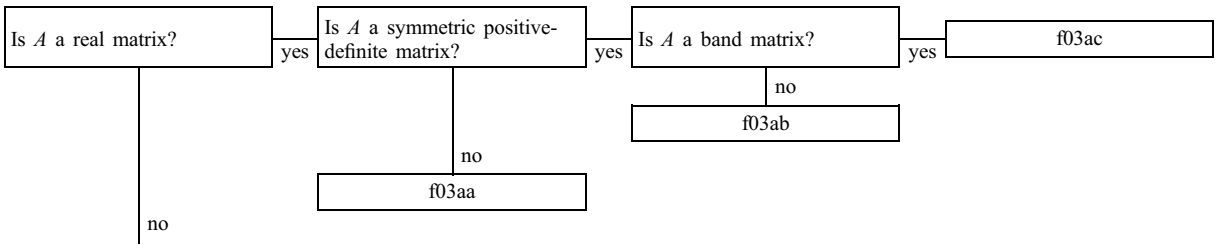
(ii) **General Purpose Functions**

These give the value of the determinant in its scaled form, $d1$ and $d2$, and also give the triangular decomposition of the matrix A in a form suitable for input to either the inversion functions of Chapter F01 or the solution of linear equation functions in Chapter F04.

4 Decision Tree

Tree 1

Note: if at any stage the answer to a question is 'Don't know' this should be read as 'No'.



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6 References

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Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag